ON DYNAMICAL ADJUSTMENT MECHANISMS FOR THE COSMOLOGICAL CONSTANT *

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After recalling why dynamical adjustment mechanisms represent a particularly attractive possibility for solving the cosmological constant problem, we briefly discuss their intrinsic difficulties as summarized in Weinberg's no-go theorem. We then comment on some problems of the recently proposed 'self-tuning' mechanism in 4+1 dimensions. Finally, we describe an alternative approach which uses the time-evolution of the universe to achieve a dynamical relaxation of the cosmological constant to zero.

We are used to describing particle physics and cosmology by an appropriate low-energy effective field theory. For energies presently accessible in the laboratory, this effective field theory contains the quantum fields of the standard model together with classical gravity. In this framework, one expects the vacuum fluctuations of the quantum fields to produce a cosmological constant corresponding to an energy density $\sim \mu^4$, where the cutoff μ is at least $\mathcal{O}(\text{TeV})$. This has to be compared with the experimental upper bound $\sim \mathcal{O}(\text{meV}^4)$. It is hard to imagine that this discrepancy can be resolved by any high-energy symmetry argument, since numerous low-energy standard-model contributions to the vacuum energy, which depend on all the details of the low-energy field theory, exist. An example is provided by the gluon condensate of QCD, the value of which depends on the non-perturbative dynamics. Even small variations of this condensate, associated, e.g., with the light quark masses, are sufficient to exceed the experimental upper bound on the cosmological constant by a vast amount. It is difficult to see how a high-energy mechanism, based, e.g., on supersymmetry or string theory, would 'know' enough about all these effective low-energy parameters to compensate their contribution with the high accuracy required.

The idea of a dynamical adjustment of the cosmological constant to zero represents an attractive alternative possibility (see [1] for a review). Ideally, such a mechanism would ensure that, given the sudden appearance of a new contribution to the vacuum energy density (e.g., through a phase transition in late cosmology), an equal opposite-sign contribution is created and exponential expansion or collapse are avoided.

Unfortunately, as was explained in [1], the most straightforward attempts of constructing such a mechanism are bound to fail. More specifically, it was shown that one can not build an action for gravity, standard model and a finite number of scalar fields which would, without fine-tuning, be extremized by a space-time independent field configuration with zero curvature. A very simple, intuitive argument can be based on the action

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} M^2 R + \mathcal{L}_{SM} + \mathcal{L}(\phi) \right) , \tag{1}$$

which, after integrating out the standard model degrees of freedom and restricting oneself

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to constant fields ϕ, takes the form

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} M^2 R - \Lambda_{SM} - V(\phi) \right) . \tag{2}$$

It is now clear that one can not design a potential $V(\phi)$ such that the minimum value of $\Lambda_{SM} + V(\phi)$ is zero for any value of Λ_{SM}. Furthermore, the situation is not improved by generalizing the action according to $\Lambda_{SM} \rightarrow \Lambda_{SM} f(\phi)$, as would be the case in Brans-Dicke-like theories*. As discussed in more detail in [1], the reason for these difficulties lies in the fact that, for constant fields, the action depends on the metric only through an overall factor $\sqrt{-g}$. Thus, the problem is reduced to the requirement that the minimal value of a potential is zero. Given the additive contribution Λ_{SM}, this leads to fine-tuning.

To design a working adjustment mechanism, one needs to relax some of the assumptions of the above no-go theorem. For example, the low energy effective theory of gravity could be quite different from Einstein's general relativity without conflict with experiment. This has been demonstrated very impressively in [2], where a non-compact extra dimension gives rise to a continuum of gravitational Kaluza-Klein modes without a mass gap (the Randall-Sundrum II scenario). Indeed, starting with [3], many attempts have been made to construct an adjustment mechanism in this framework.

The original model is based on gravity + a scalar field in 4+1 dimensions, with the standard model fields being localized on a 3+1 d boundary (with orbifold boundary conditions). After integrating out the standard model degrees of freedom, the action is

$$S = \int d^4x dy \sqrt{-g_5} \left(\frac{1}{2} M_5^3 R - \frac{3}{2} (\partial\phi)^2 \right) - \int d^4x \sqrt{-g_4} \Lambda_{SM} \exp \left(\frac{2\phi}{M_5^{3/2}} \right) . \tag{3}$$

The essential observation is that, for any value of Λ_{SM}, the equations of motion derived from this action have a static solution with vanishing brane curvature. In the bulk, this solution has a curvature singularity at finite proper distance from the brane. This gives rise to the hope that low-energy 4d gravity will result since the fifth dimension is effectively finite.

The above 'self-tuning' scenario has already been criticized on a rather fundamental level in [4]. Nevertheless, we want to describe an objection to [3] which, although close in spirit to [4], might be somewhat simpler and more direct. The point is that the field configuration of [3] does not extremize the action Eq. (3). This is immediately clear since the only non-derivative coupling of ϕ appears in the factor multiplying Λ_{SM}. Thus, for nonzero Λ_{SM}, changing ϕ by an infinitesimal constant leads to a change of S proportional to that constant.

All this is not in contradiction to the claim of [3] that their field configuration solves the equations of motion (everywhere outside the singularity). The reason is that, when varying an action on a finite interval (as is appropriate if a singularity is present), one gets, in addition to the equations of motion, a boundary term, which has not been considered in [3]. Therefore it appears that one either has to give up the action principle or to allow for the possibility of a non-derivative coupling of ϕ to the physics at the singularity. In the latter case, it seems likely that some form of fine-tuning will, after all, be required.

In the remainder of this paper, we want to outline a different way of relaxing the assumptions of Weinberg's no-go theorem [5,6]. In this approach, one does not insist on a

* One has to require that $[\Lambda_{SM} f(\phi) + V(\phi)]' = 0$ implies $\Lambda_{SM} f(\phi) + V(\phi) = 0$. This leads to the condition $f(\phi) = \text{const.} \times V(\phi)$, which does not solve the original problem.

flat universe as an extremum of the action, but satisfies oneself with a cosmology that approaches the flat geometry asymptotically as the universe grows old.

A possible mechanism for such a 'time-dependent' adjustment of the cosmological constant has been suggested in Rubakov's scenario of a relaxation at inflation [5]. Developing his ideas, we present a time-dependent adjustment mechanism for the cosmological constant that can be at work in a realistic, late Friedmann-Robertson-Walker universe [6].

In our model, the energy density is dominated by non-standard-model dark matter together with a quintessence field φ. As in Rubakov's scenario, the cosmological constant, characterized by a field χ, rolls down a potential and approaches zero asymptotically. This is realized by a kinetic term for χ that diverges as t^4 at large t. The effective t^4 behaviour is realized with the help of φ. The asymptotic stability of this solution is ensured by the coupling to a Brans-Dicke field σ.

The action of our model can be decomposed according to

$$S = S_E + S_{SF} + S_{SM}\,, \tag{4}$$

where S_E is the Einstein action, S_{SF} the scalar field action, and S_{SM} the standard model action, which is written in the form

$$S_{SM} = S_{SM}[\psi, g_{\mu\nu}, \chi] = \int d^4x\sqrt{-g}\,\mathcal{L}_{SM}(\psi, g_{\mu\nu}, \chi)\,. \tag{5}$$

Here ψ stands for all standard-model fields. The scalar χ is assumed to govern the effective UV-cutoffs of the different modes of ψ, thereby influencing the effective cosmological constant. Units are chosen such that $16\pi G_N = 1$.

Integrating out the fields ψ, one obtains (up to derivative terms)

$$S_{SM} = \int d^4x\sqrt{g}\,V(\chi)\,. \tag{6}$$

Let the potential $V(\chi)$ have a zero, $V(\chi_0) = 0$ with $\alpha = V'(\chi_0)$, and rename the field according to $\chi \to \chi_0 + \chi$. Then the action near $\chi = 0$ becomes

$$S_{SM} = \int d^4x\sqrt{g}\,\alpha\chi\,. \tag{7}$$

Due to this potential the field χ will decrease (for $\alpha > 0$) during its cosmological evolution. It can be prevented from rolling through the zero by a diverging kinetic term [7].

First, let the geometry be imposed on the system, i.e., assume a flat FRW universe with $H = (2/3)\,t^{-1}$. With a kinetic lagrangian

$$\mathcal{L}_{SF} = \frac{1}{2}\,\partial^\mu\chi\partial_\mu\chi\,t^4\,, \tag{8}$$

one finds a solution where $\chi = (\alpha/6)\,t^{-2}$, which provides an acceptable late cosmology.

Clearly, it requires fine tuning of the initial conditions to achieve the desired behavior $\chi \to 0$ as $t \to \infty$. However, this fine tuning can be avoided by adding a Brans-Dicke field σ that 'feels' the deviation of χ from zero and provides the appropriate 'feedback' to the kinetic term so that χ reaches zero asymptotically independent of its initial value.

The field σ has a canonical kinetic term and it is coupled to $\mathcal{L}_{SM}$ by the substitution $g_{\mu\nu} \to g_{\mu\nu}\sqrt{\sigma}$ in Eq. (5),

$$S_{SM} = S_{SM}[\psi, g_{\mu\nu}\sqrt{\sigma}, \chi] = \int d^4x\,\sigma\sqrt{g}\,\mathcal{L}_{SM}(\psi, g_{\mu\nu}\sqrt{\sigma}, \chi)\,. \tag{9}$$

Integrating out the fields ψ, one obtains

$$S_{SM} = \int d^4x \sqrt{g}\, \alpha\sigma\chi \tag{10}$$

near $\chi = 0$. The scalar field lagrangian is now taken to be

$$\mathcal{L}_{SF} = \frac{1}{2}(\partial\chi)^2\sigma^2 t^4 + \frac{1}{2}(\partial\sigma)^2 - \beta\sigma t^{-2}\,. \tag{11}$$

This gives rise to the asymptotic solution $\chi = \chi_0\, t^{-2}$ and $\sigma = \sigma_0 =$ const. This solution is stable, i.e., one finds the same asymptotic behaviour for a range of initial conditions. The stability does not depend on the precise values of the parameters α and β.

What remains to be done is the replacement of the various explicit t-dependent functions by the dynamics of an appropriate field. This can be achieved using the simplest version of quintessence [8] with an exponential potential $V_Q(\varphi) = e^{-a\varphi}$. One finds the late-time behaviour $\varphi = (2/a)\ln t$. The explicit time dependence in Eq. (11) can now be replaced by a coupling to φ. Technically, this is realized by the substitution $t^2 \to e^{a\varphi}$ in $\mathcal{L}_{SF}$.

The complete lagrangian, including the curvature term and the effective standard model action, Eq. (10), reads

$$\mathcal{L} = R + \frac{1}{2}(\partial\chi)^2\sigma^2\, e^{2a\varphi} + \frac{1}{2}(\partial\sigma)^2 + \frac{1}{2}(\partial\varphi)^2 + \alpha\sigma\chi + (1-\beta\sigma)\, e^{-a\varphi}\,. \tag{12}$$

We have checked numerically that the resulting late time solution, which gives rise to an acceptable cosmology, is stable with respect to small variations of all parameters and initial conditions. The only problem is the too strong coupling of the Brans-Dicke field to baryons. However, this can probably be avoided in a more carefully constructed model or by the ad-hoc introduction of a kinetic term for σ that grows for large t.

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